13ème congrès de mécanique (CMM2017)
11-14 Avril 2017 – Meknès

On the generalization of contour integral in three-dimensional crack problem

EL KABIR Soliman[1,2], MOUTOU PITTI Rostand[2,3], DUBOIS Frederic[1], RECHO Naman[2,3], LAPUSTA Yuri[3,4]

[1]Université de Limoges, Centre du génie Civil, GEMH, 19300 Egletons, France
[2]Université Clermont Auvergne, Institut Pascal, BP 10448, 63000 Clermont-Ferrand, France
[3]CNRS, UMR 6602, Institut Pascal, 63171 Aubière, France
[4]Institut Français de Mécanique Avancée (IFMA), Institut Pascal, 63171 Aubière, France

Contacts : soliman.el-kabir@unilim.fr

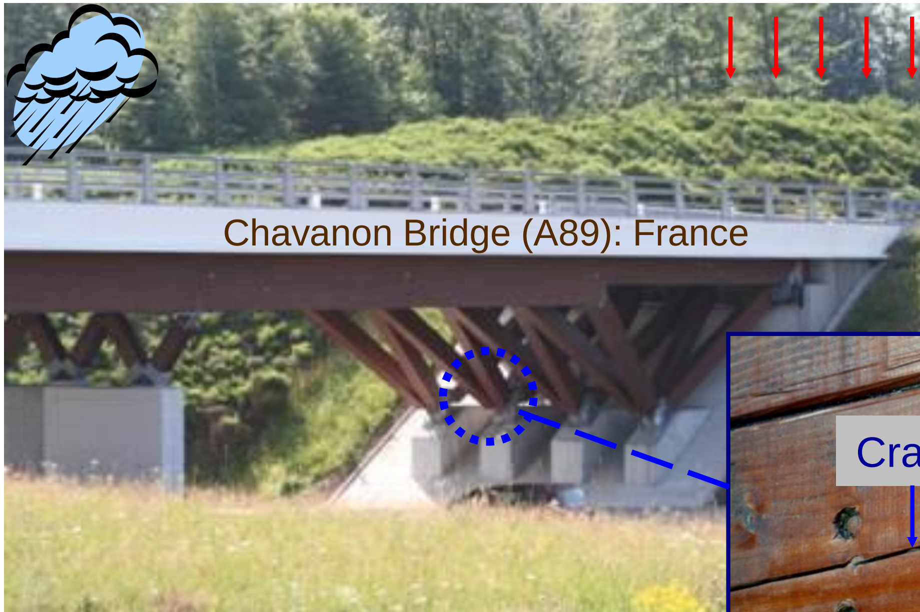
Scientific context

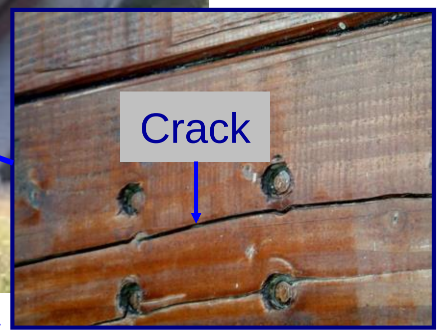
- Predicting the behavior of wood structures under mixed mode loading
- Developing specific tools for three-dimensional configurations
- Consider thickness effect under variable environments
- Need for applications related to the inspection and diagnosis of structures

- Numerical development of the J-integral concept for 3D problems.
- The generalization toward its G_θ^{3D} implementation form
- Several numerical applications are proposed
- The efficiency of the proposed integral is compared

Scientific context

Climatic loading Moisture variation Creep loading



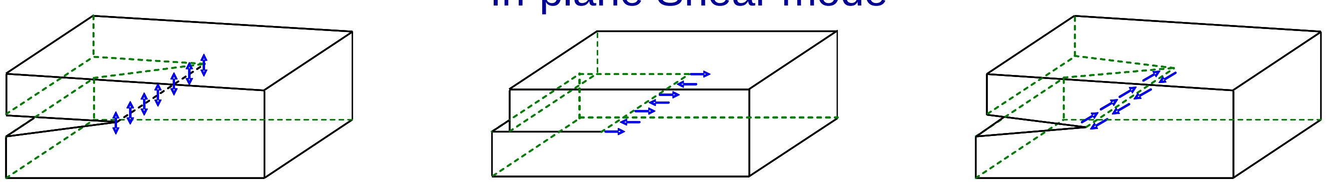


Orthotropic behavior and viscoelastic effects

Opening mode

In-plane Shear mode

Out-of-plane shear mode



Mode I Mode II Mode III

Modes of fracture

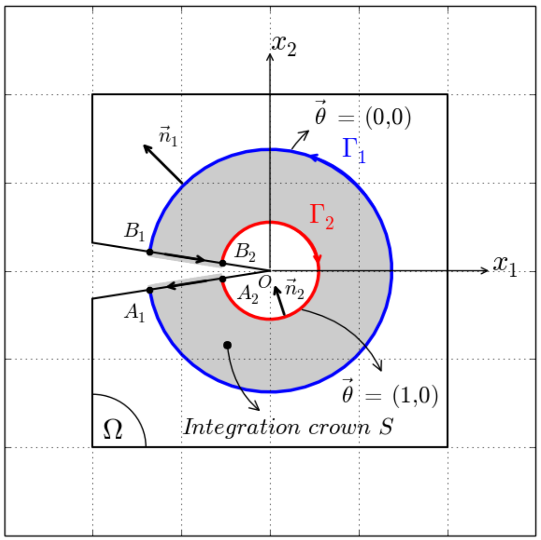
Plan

- ✓ Scientific context
- ✓ Analytical formulation (J^{3D}-integral, G_{θ}^{3D} , …)
- ✓ Numerical implementation
- ✓ Conclusions and outlooks

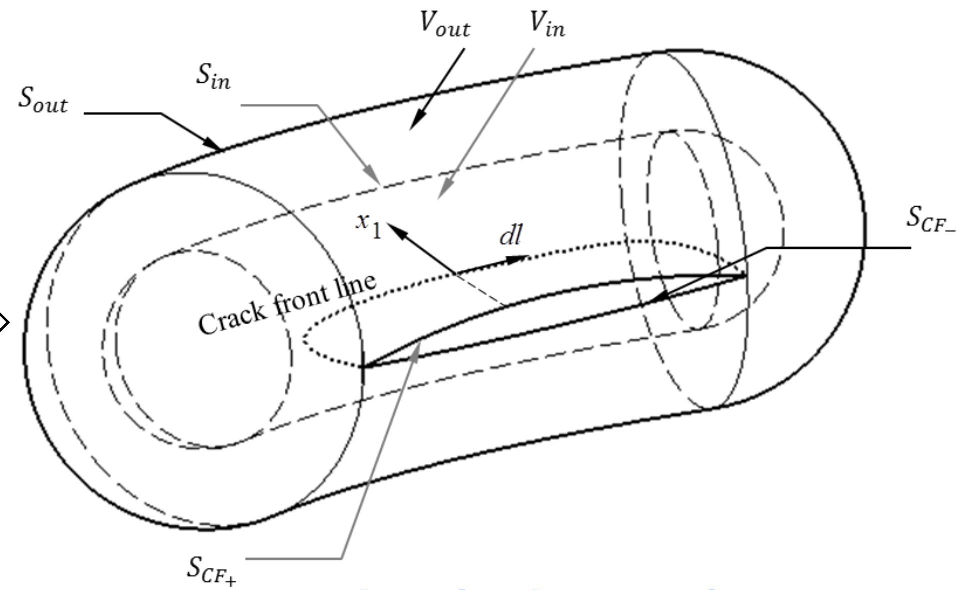
J-integral in 3D configurations

Rice's integral

$$J^{2D} = \int_{\Gamma} \left(W.n_1 - \left(\sigma_{ij}.n_j.u_{i,1} \right) \right).d\Gamma$$



Integration domain size for 2D



3D Closed volume and surfaces integration domains

The main advance of this form is the presence of an arbitrary crack front line enclosed by a 3D surface

J^{3D} -integral description

The J-integral formulation is based on the Noether's theorem application (Noether 1918) :

$$\delta L = \int_V \int_t \delta W . dV . dt = 0 \quad \Longleftrightarrow$$

A Gauss-Ostrogradski transformation allows us writing the Lagrangian's invariance in the form:

$$\int_S \left(W . n_k - \left(\sigma_{ij} . n_j . u_{i,k}\right)\right) . dS + \int_V \left(\sigma_{ij} . \left(\varepsilon_{ij}\right)_{,k} - W_{,k}\right) . dV = 0$$

$$J^{3D} = \int_{S_{\Gamma_{out}}} \left(W . n_1 - \left(\sigma_{ij} . n_j . u_{i,1}\right)\right) . dS - \int_{S_{cr}} \sigma_{ij} . u_{i,1} . n_j . dS - \int_{V_{\Gamma_{out}}} \left(W_{,1} - \sigma_{ij} . \left(\varepsilon_{ij}\right)_{,1}\right) . dV$$

1 Stationary crack

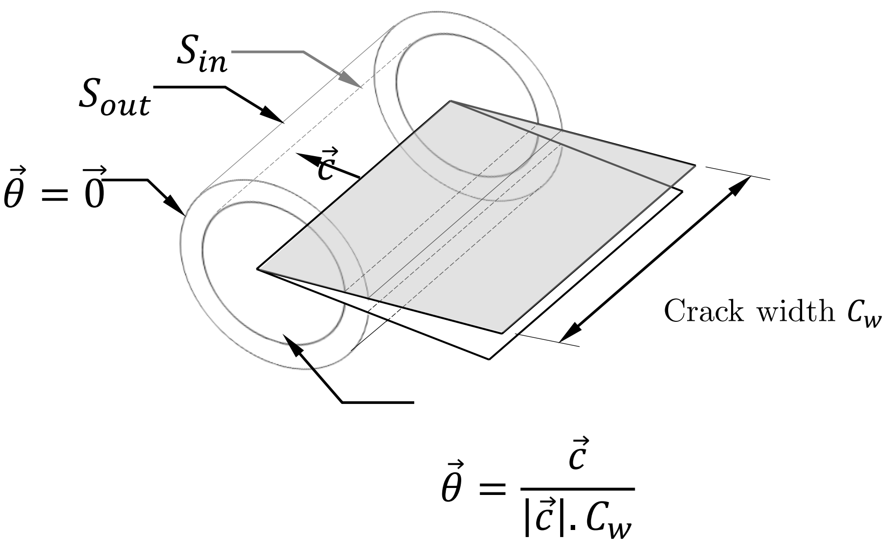
2 Crack lips pressure

3 Crack propagation

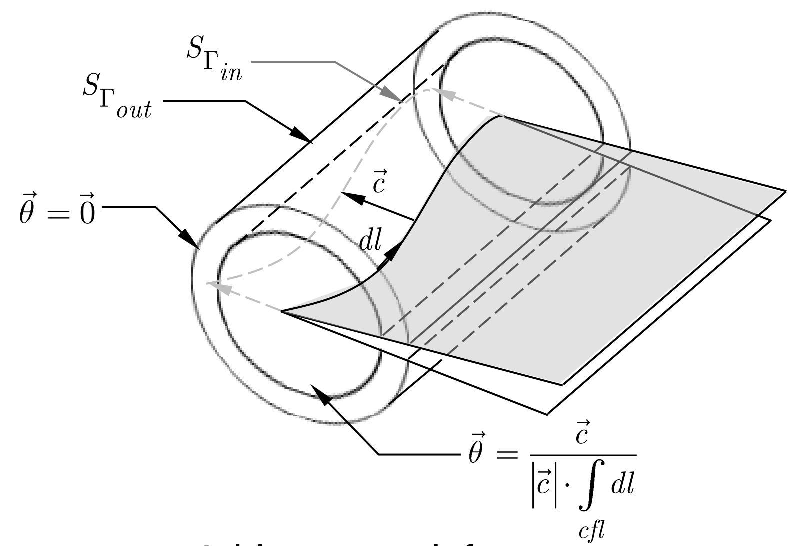
G_θ^{3D} Integral

$$G_\theta = \int_V (P_{kj}.\theta_{k,j}).n_j.dV + \int_{S_{CF}} \sigma_{ij}.u_{i,k}.n_j.\theta_k.dS - \int_{V_{\Gamma_2}} \left(W_{,k} - \sigma_{ij}.(\varepsilon_{ij})_{,k}\right).\theta_k.dV$$

① ② ③



Linear crack front



Arbitrary crack front

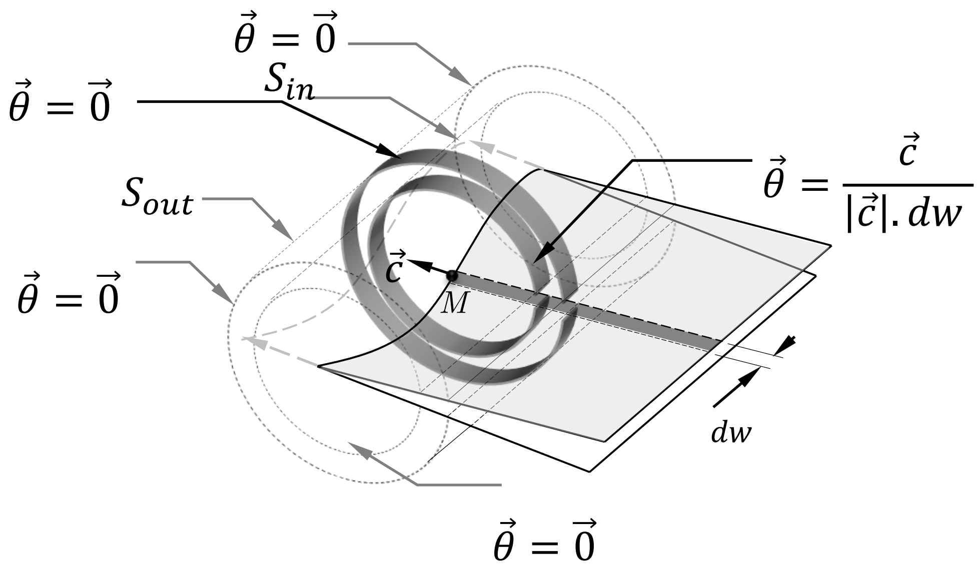
$$\vec{\theta} = \frac{\vec{c}}{|\vec{c}|.C_w} \text{ on } S_{\Gamma_{out}} \text{ and } \vec{\theta} = 0 \text{ on } S_{\Gamma_{in}}$$

$$\vec{\theta} = \frac{\vec{c}}{|\vec{c}|.\int_{cfl} dl} \text{ on } S_{\Gamma_{out}} \text{ and } \vec{\theta} = 0 \text{ on } S_{\Gamma_{in}}$$

$G(M)$ integral

$$G_\theta = \int_V (P_{kj}.\theta_{k,j}).n_j.dV + \int_{S_{CF}} \sigma_{ij}.u_{i,k}.n_j.\theta_k.dS - \int_{V_{\Gamma_2}} \left(W_{,k} - \sigma_{ij}.\left(\varepsilon_{ij}\right)_{,k}\right).\theta_k.dV$$

① ② ③



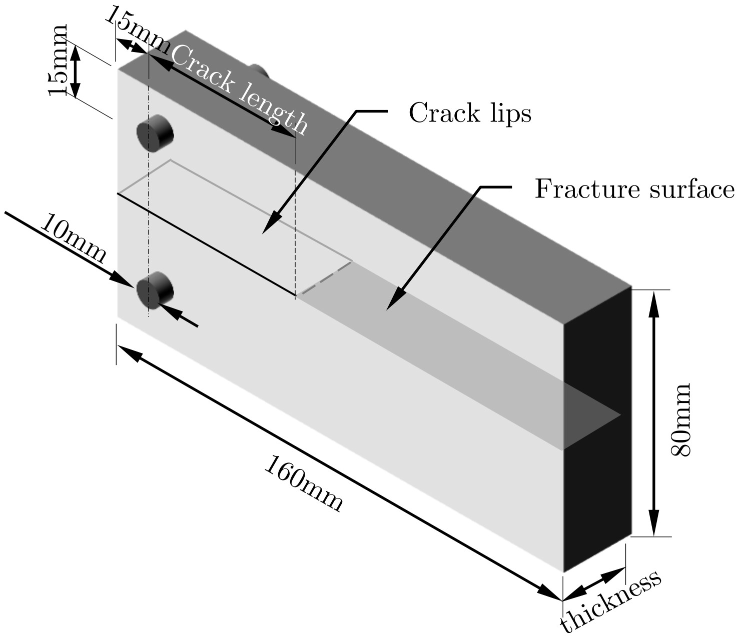
Definition of $\vec{\theta}$ around a close crown

$$\vec{\theta} = 0 \; on \; S_{\Gamma_{out}}, \vec{\theta} = \frac{\vec{c}}{|\vec{c}|.dw} \; on \; C \cap S_{\Gamma_{out}} \; and \; \vec{\theta} = 0 \; on \; S_{\Gamma_{in}}$$

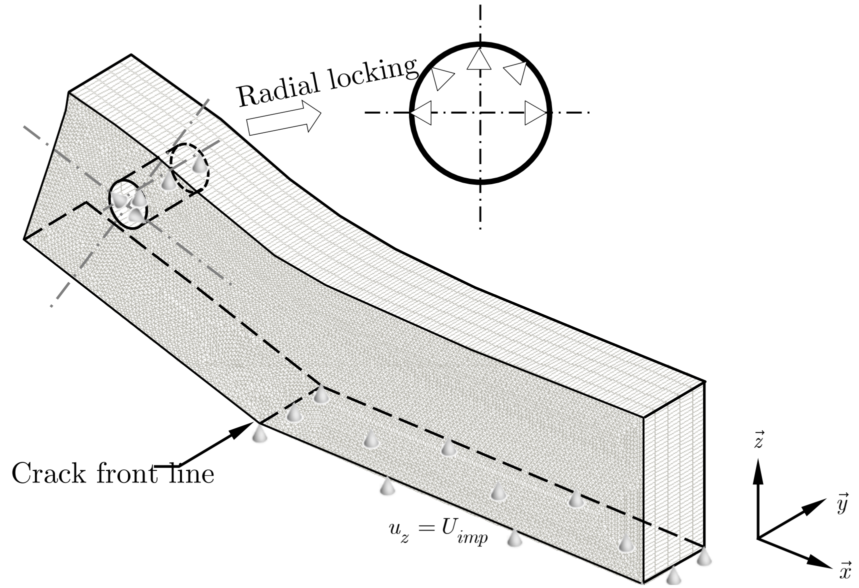
The average energy release rate can be calculated with an integration of along the crack front line divided by the crack width

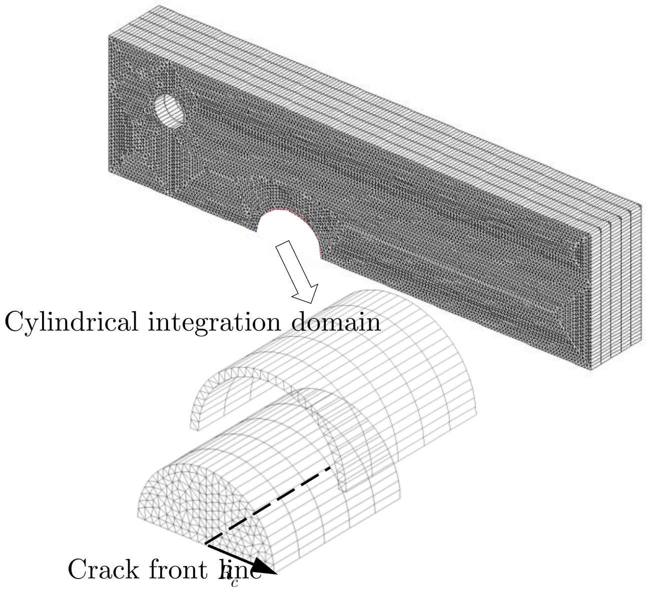
Numerical validation

The finite element implementation is based on a Double Cantilever Beam loaded in an open mode.



Specimen DCB

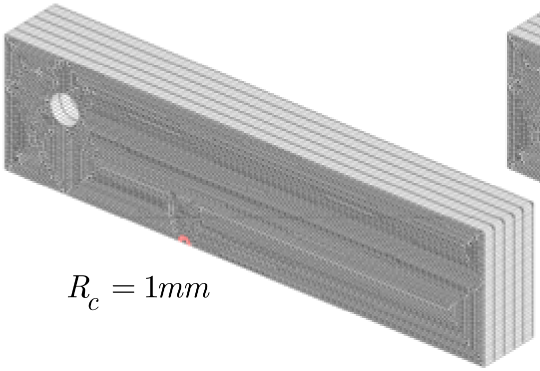


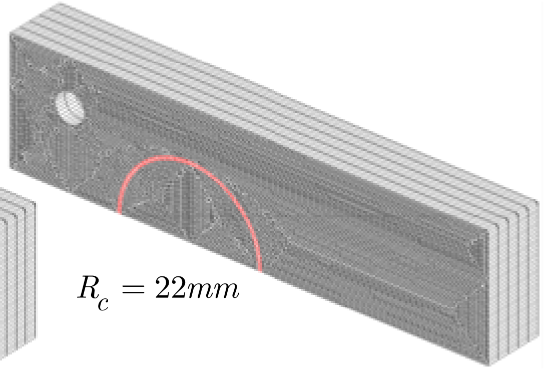


Steel material characterized by :

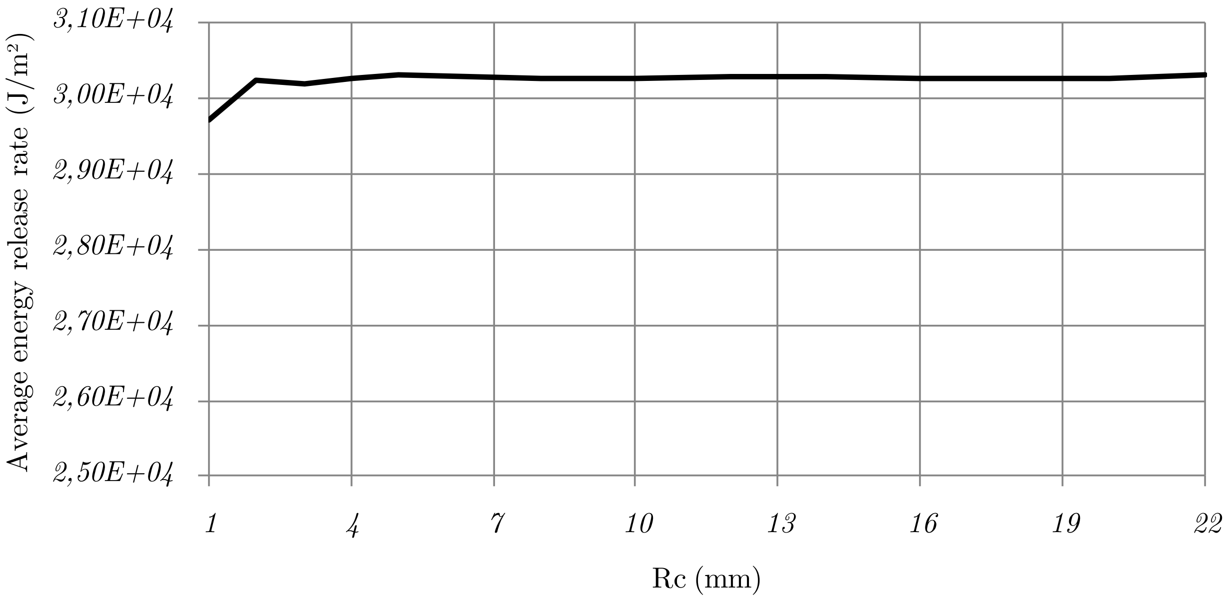
- Thickness =20 mm
- Linear crack front line
- $E = 210MPa,\ \nu = 0{,}3$

Numerical validation





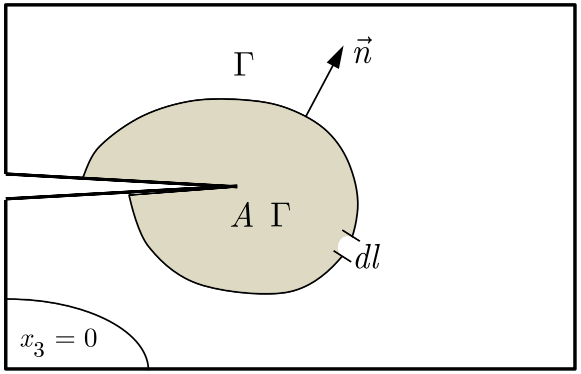
We can shows the variations of the energy release rate versus R_c:



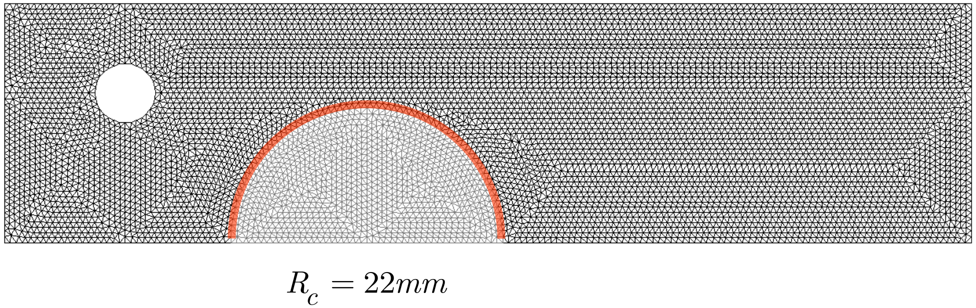
Numerical results validate the non-dependence of the integration domain with an average value of 30.3kJ/m²

Surface integration domains for the Bui's integral :

$$J_{Am} = \underbrace{\int_{\Gamma}\Big(W.n_1 - (\sigma_{ij}.n_j.u_{i,1})\Big).d\Gamma}_{J^{2D}} - \int_{A(\Gamma)} \frac{d}{dx_3}(\sigma_{i3}.u_{i,1}).dA(\Gamma)$$



Integration domains

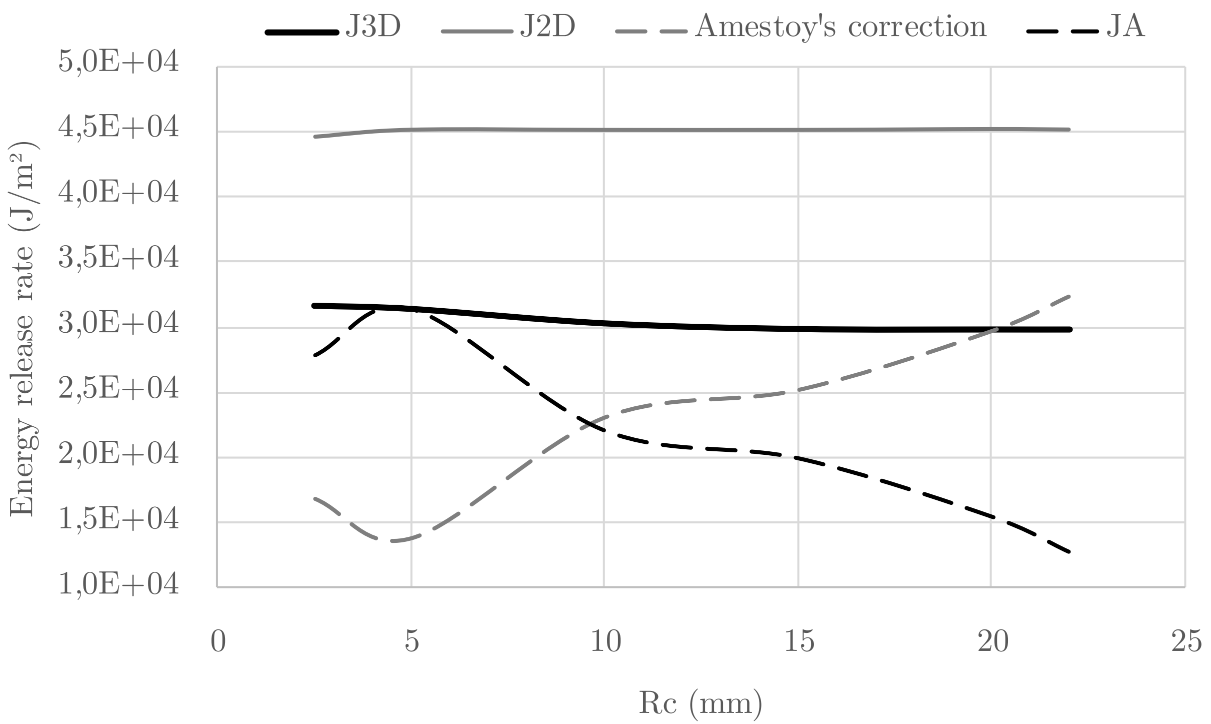


Integration domain size for 2D model

Physical interpretation

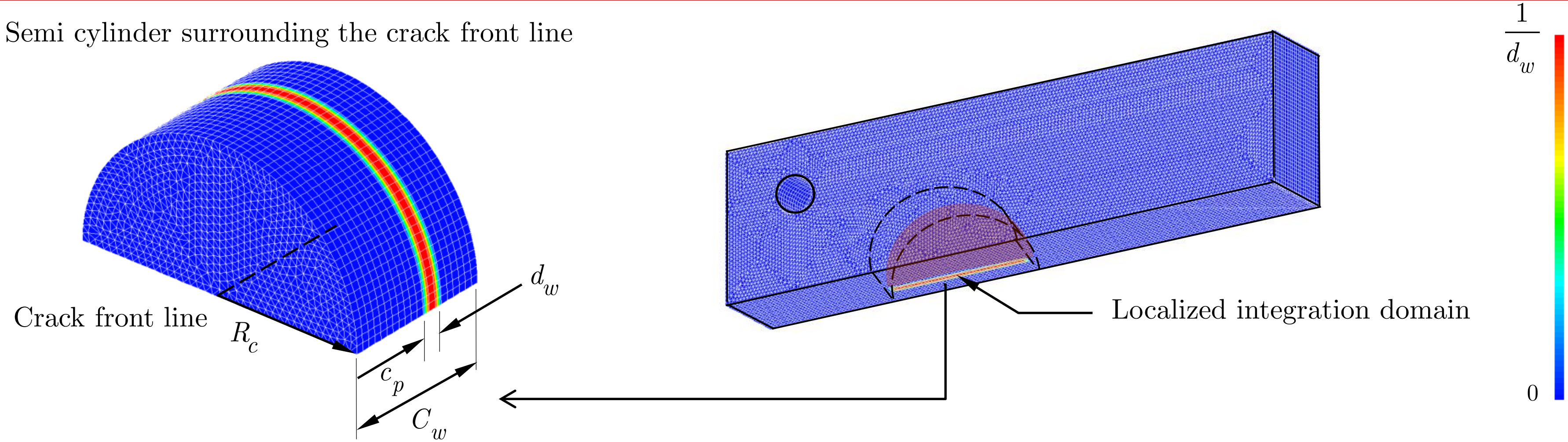
Surface integration domains for the Bui's integral :

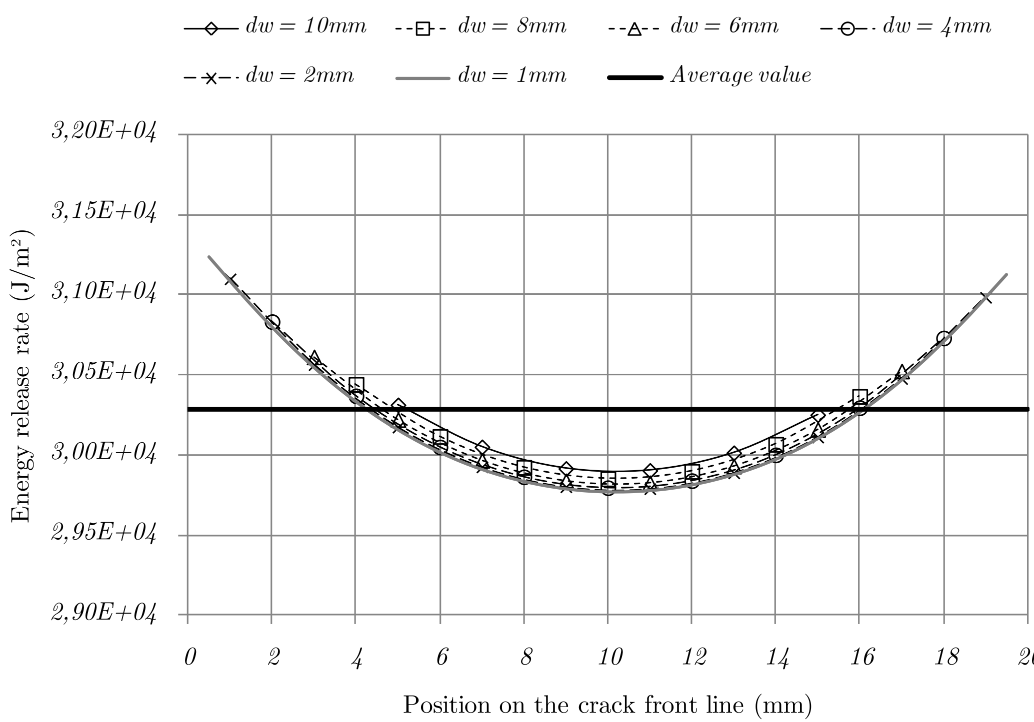
$$J_{Am} = \underbrace{\int_{\Gamma}\left(W.n_1 - (\sigma_{ij}.n_j.u_{i,1})\right).d\Gamma}_{J^{2D}} - \int_{A(\Gamma)}\frac{d}{dx_3}(\sigma_{i3}.u_{i,1}).dA(\Gamma)$$



Comparison between J^{2D} and J^{3D} approaches

Numerical validation





Conclusions and outlooks

- J-integral parameter adapted to three-dimensional problems
- Numerical validation of the J-integral
- The definition of the average energy release rate distribution along the crack front
- Non-dependence integration domain characterizing invariant integral concept.
- Comparison with classical Amestoy's integral

- Generalization of the M-integral for more complex problems
- Application to a M-integral applied for three dimensional
- Generalization of the local mechanical fields
- Experimental strain analysis using the image correlation method

13ème congrès de mécanique (CMM2017)
11-14 Avril 2017 – Meknès

On the generalization of contour integral in three-dimensional crack problem

EL KABIR Soliman[1,2], MOUTOU PITTI Rostand[2,3], DUBOIS Frederic[1], RECHO Naman[2,3], LAPUSTA Yuri[3,4]

[1]Université de Limoges, Centre du génie Civil, GEMH, 19300 Egletons, France
[2]Université Clermont Auvergne, Institut Pascal, BP 10448, 63000 Clermont-Ferrand, France
[3]CNRS, UMR 6602, Institut Pascal, 63171 Aubière, France
[4]Institut Français de Mécanique Avancée (IFMA), Institut Pascal, 63171 Aubière, France

Contacts : soliman.el-kabir@unilim.fr