# Non-destructive measurement of orthotropic elastic properties of wood samples by their modal impulse response

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## Context and objectives

Wood represents a major class of versatile materials in mechanics, comparable to metals on various criteria such as annual world production tonnage. The density of wood is 3 to 15 times lower than that of metals, due to its cellular structure, whose walls are essentially made up of layers of long-fiber nano-composites. This makes wood one of the most efficient materials for many applications. However, the knowledge of the transverse ($E_R$ radial modulus and $E_T$ tangential modulus) and shear ($G_{RT}$, $G_{LT}$ and $G_{LR}$) elastic properties of this orthotropic, heterogeneous, hygroscopic and variable material are still limited due to a lack of rapid and efficient characterization tools and methods. Currently, the existing technical data in databases are incomplete for material selection since, generally, only the longitudinal elastic modulus is available, whereas the other elastic properties are essential for high-end and high-performance applications.

The aims of this study are: to rapidly estimate as many elastic parameters as possible from a single wood sample using the modal analysis of its vibrational impulse response; to investigate the damping characteristics and relate them to viscous behaviour; to enrich the woods database with orthotropic elastic constants; and to analyze the correlations between macroscopic viscoelastic behavior and ultrastructural parameters, including density and microfibril angle (Al Fay et al. 2022).

## Material and methods

### Samples

The wood samples used in this study, specifically Paulownia and Beech, all had dimensions of 130×60×10 mm³ and were taken from CIRAD Xylothèque (Langbour et al. 2019), an extensive collection of wood species (over 34 000 samples of 8 400 species). Additionally, two other materials, polystyrene (PS) and a unidirectional glass fiber/epoxy matrix composite, were also investigated with the same sample dimensions, serving as a fundamental step towards mastering the essential tools required to study more complex materials, such as wood.

### Experimental set-up

The method used is a non-destructive test by impulse modal analysis, known for its efficiency to allow a large number of elastic constants to be determined from a single sample (Longo et al. 2018). To achieve free-free boundary conditions, all samples were mounted on soft elastic supports. Vibrations were induced in the sample using either a light hammer or a tiny steel ball. The oscillations were measured using either a high-sensitivity microphone (PCB - 130F21), with a frequency range from 10 Hz to 20 kHz, or an accelerometer (PCB - 352A26), with a frequency range from 2 Hz to 10 kHz and a cut-off frequency of 30 kHz. Signal acquisition was carried out using a suitable device capable of simultaneously handling up to 4 channels (NI-4431). Finally, the data acquired were analysed using ModalVIEW software to extract the modal parameters, including the resonance frequencies and the corresponding damping factors.

### *Test configurations*

Different configurations were tested for each sample, so called "out of plane" and "in plane", representing the direction of impact and sensor measurement. For each configuration, different positions were considered for both the microphone or accelerometer and the impact, which had two primary objectives. First, we aimed to measure the first resonance frequencies and their associated damping exhaustively. Secondly, we needed to link each measured resonance frequency with its corresponding computed mode, based on the relative positions of the impact and the microphone in relation to the computed vibration nodes using the initial set of elastic constants (see below). This link is crucial for accurately matching measured resonance frequencies with calculated ones.

### *Inverse identification of elastic constants*

The inverse identification process of the elastic properties from the resonance frequencies is an adaptation of the one described in Longo et al. (2018). First of all, the stiffness matrix coefficients need to be initialised. This was done for polystyrene using data from Yadav's thesis (Yadav 2019). These values were determined through dynamic mechanical analysis (DMA) method. For the glass fiber/epoxy composite material, the initial values were calculated using the Mori-Tanaka homogenization scheme in Digimat software[2]. For wood, we started by initializing the stiffness matrix coefficients using the sample density and the empirical relationships from Guitard and El Amri (1987). Subsequently, modal shapes and frequencies are computed using Cast3m FEA software[3]. This model is capable of considering the orthotropy of wood and the disorientation of rings (or even of the grain) in the sample with a cylindrical coordinate system. Additionally, we computed resonant frequencies and modal shapes faster by using a Resonant Ultrasound Spectroscopy (RUS) program (Fig 2008), using a Rayleigh-Ritz method with each component of the displacement expand in a Cartesian power series, thus taking into account the orthotropy of wood but in a Cartesian coordinate system (flat rings parallel to one of the lateral faces of the samples, grain in the longitudinal direction of the samples). The computed results are then associated with the measured resonance frequencies using the different test configurations explained above. A preliminary parametric sensitivity analysis, of the frequency of each mode relatively to the different elastic constants, is performed to obtain a sensitivity matrix. This analysis serves two main purposes: firstly, to identify which elastic constants can be reliably obtained, ensuring more robust results; and secondly, to conduct multi-step minimization. This approach is based on an iterative process to optimize elastic constants by minimizing the least squares error between experimental and computed resonance frequencies. Additionally, a matching and tracking of eigenmodes during the identification process is done.

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[2] https://hexagon.com/products/digimat

[3] http://www-cast3m.cea.fr/

## Results

### *Polystyrene (isotropic case)*

After calculating the sensitivity matrix, we identified polystyrene's elastic constants through various mode combinations, initially considering all 13 measured modes with frequencies ranging from 1.413 to 8.834 kHz, and then only 2 close modes at a time, each sensitive to E (Young's modulus) or G (shear modulus). The results indicated that it is feasible to identify the elastic constants (E, G, and subsequently ν) using only 2 modes, one of bending and the other of torsion. Furthermore, we observed that the frequency range of the selected modes influences the values of the “apparent” elastic constants. This effect is attributed to the viscoelastic properties of the material. Moroever, using an orthotropic model with 13 measured frequencies, slight differences were observed in Young's and shear moduli across different directions. This suggests a small anisotropy that may be attributed to the manufacturing process of the sample.

### *Unidirectional glass fiber/epoxy matrix composite (transverse isotropic case)*

The study initially used 11 measured frequencies (ranging from 2.055 to 19.055 kHz) to calculate a sensitivity matrix. The results revealed that several modes were highly sensitive to three elastic constants $E_1$, $E_3$, and $G_{13}$ (3 being the longitudinal direction of the sample and 1 its width). However, some modes showed only moderate sensitivity to $G_{23}$ and $G_{12}$, and none were sensitive to $E_2$ due to the small plate thickness. All the measured frequencies were almost not sensitive to the three Poisson’s ratios due to the sample's dimensions (Lauwagie et al. 2010) among others. However, our goal remained the successful identification of all five elastic constants ($E_1$, $E_3$, $G_{13}$, $G_{23}$, and $G_{12}$). To achieve this, we conducted the identification process using only five frequencies, selecting the most sensitive modes to each of these elastic constants. Results showed that the elastic constants of the unidirectional composite can be calculated with only these 5 frequencies. Furthermore, we intend to explore the optimal number of frequencies for determining elastic constants in greater detail during this thesis. The damping factor associated to the bending modes in the longitudinal direction 3 (parallel to the glass fibers) was observed to be lower than the ones of torsion modes or bending modes in the transverse direction. This difference can be attributed to the higher viscosity of the epoxy matrix compared to the glass fibers. In fact, the matrix is more stressed than the glass fibers in torsion and transverse bending while glass fibers are most stressed in the longitudinal bending mode.

### *Beech (Fagus sylvatica) and Paulownia (Paulownia tomentosa) (orthotropic case in cartesian and cylindrical coordinate system, in progress)*

Our objective was to identify the elastic constants of Beech and Paulownia 130×60×10 $mm^3$ samples. To account for the curvature and possible disorientation of growth rings relatively to the lateral faces of the wood samples, we mathematically estimated the position of the pith of the tree using scanned images of the rings on both cross-section faces of the plate and use it in Cast3m FEA software for the numerical modal analysis. For Paulownia samples exhibiting growth rings roughly flat and parallel to the thickness (quarter-sawn samples), we can use the RUS program (Fig 2008) for the modal analysis too and compare the results with the use of Cast3m. We conducted an analysis by computing the sensitivity matrix using data from 5 measured frequencies (ranging from 1.732 to 8.301 kHz). Our findings indicate that these modes exhibit a high sensitivity, exceeding 30%, to three specific elastic constants $E_R$ (radial direction), $E_L$ (longitudinal direction), and $G_{RL}$ (radial-longitudinal plane). This high sensitivity allows us to confidently identify these elastic constants. Additionally, there were a moderate sensitivity, ranging from 5% to 30%, to the elastic constants $G_{TL}$ (tangential-longitudinal plane) and $G_{RT}$ (radial-tangential plane). Our findings demonstrate that the RUS program has proved to be very time-efficient effective in determining the elastic constants of wood samples that can be analysed within a Cartesian coordinate system. In the analysis of beech samples, for quarter-sawn wood with flat, parallel growth rings, two elastic constants ($E_L$ and $G_{RL}$) can be confidently identified, but there is moderate sensitivity in resonance frequencies to three other constants ($E_R$, $G_{TL}$, and $G_{RT}$). In contrast, in flat-sawn samples with moderately curved rings parallel to the width, three specific elastic constants ($E_T$, $E_L$, and $G_{TL}$) can be confidently identified, with moderate sensitivity to the remaining two ($G_{RL}$ and $G_{RT}$) based on resonance frequencies.

## Conclusion

This study has provided valuable insights into the non-destructive measurement of the orthotropic elastic properties of wood samples through modal impulse response analysis. By exploring different materials, including polystyrene and a unidirectional glass fiber/epoxy matrix composite, our results have demonstrated the effectiveness of this method in identifying most of the elastic constants. Furthermore, for Paulownia and Beech wood samples, the study has shown that the method is capable of distinguishing variations in some elastic constants based on different sawing techniques. Further experiments, in particular using ultrasound, will validate these results and aim to identify the effect of the curvature of the rings and their orientation in the cross-section on the identifiable elastic constants.

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## Referencess

Al Fay A., Jullien D., Corn S., Arnould O., Langbour P. (2022) Caractérisation par analyse vibratoire des propriétés viscoélastiques d’échantillons de bois dans leur diversité naturelle. 11èmes Journées du GDR Sciences du Bois, Nice.

Fig M. (2008) Resonant ultrasound spectroscopy (RUS). Matlab® Central. http://fr.mathworks.com/matlabcentral/fileexchange/11399-resonant-ultrasound-spectroscopy--rus.

Guitard D., El Amri F. (1987) Modèles prévisionnels de comportement élastique tridimensionnel pour les bois feuillus et le bois résineux. Annales des Sciences Forestières, 44(3), 335–358.

Langbour P., Paradis S., Thibaut B. (2019) Description of the Cirad wood collection in Montpellier, France, representing eight thousand identified species. Bois et Forêts des Tropiques, 339, 7-16. Website: https://ur-biowooeb.cirad.fr/plateformes-equipements/bois-materiau/xylotheque (last access: September 2023).

Lauwagie T., Lambrinou K., Sol H. et al. (2010) Resonant-based identification of the Poisson’s ratio of orthotropic materials. Experimental Mechanics, 50, 437–447.

Longo R., Laux D., Pagano S. et al. (2018) Elastic characterization of wood by Resonant Ultrasound Spectroscopy (RUS): a comprehensive study. Wood Science and Technology, 52, 383–402.

Yadav P. (2019) Effets du temps et effets de couplage thermomécanique dans les polymères. Thèse de Doctorat de l’Université de Montpellier.